Business 4720 - Class 22

Reinforcement Learning – Function Approximation

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This Class

What You Will Learn:

- Reinforcement Learning
  - Action-value approximation
  - Policy approximation

Based On

Richard S. Sutton and Andrew G. Barto (2018) *Reinforcement Learning – An Introduction*. 2nd edition, The MIT Press, Cambridge, MA. (SB)

`http://incompleteideas.net/book/the-book.html`

Chapters 9–13

Sudharsan Ravichandiran (2020) *Deep Reinforcement Learning with Python*. 2nd edition. Packt Publishing, Birmingham, UK.

Chapters 9–11

Resources

Implementations are available on the following GitHub repo:
`https://github.com/jevermann/busi4720-rl`

The project can be cloned from this URL:
`https://github.com/jevermann/busi4720-rl.git`

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Function Approximation

Previously

- Tabular methods only suitable for small state space and discrete actions

Now

- Approximate the state-value function v by parameterized function $\hat{v}$:

$$\hat{v}(s) = \hat{v}(s, \theta) \approx v_\pi(s)$$

- Approximate the action-value function q by a parameterized function $\hat{q}$:

$$\hat{q}(s, a) = \hat{q}(s, a, \theta) \approx q_\pi(s, a)$$

- Approximate policy π by a parameterized function $\hat{\pi}$:

$$\hat{\pi}(a|s) = \hat{\pi}(a|s, \theta) \approx \pi(a|s)$$

Function Approximation

Advantages

- Continuous states and/or actions
- Tractable problems despite large state space
- Flexible functions (linear, trees, neural networks)
- Generalization to related states
  - Changing θ changes the values of multiple states
- Applicable to partially observable problems
  - State function may not depend on complete state information

Stochastic Gradient Methods

Assume a MSE value error:

$$\bar{VE} = \sum_{s \in \mathcal{S}} \mu(s) \left[q_\pi(s, a) - \hat{q}(s, a, \theta) \right]^2$$

Follow the steepest slope ("gradient"; vector of partial derivatives) of the function to update parameters:

$$\begin{aligned} \theta_{t+1} &= \theta_t - \frac{1}{2} \alpha \nabla \left[q_\pi(S_t, A_t) - \hat{q}(S_t, A_t, \theta_t) \right]^2 \\ &= \theta_t + \alpha \left[q_\pi(S_t, A_t) - \hat{q}(S_t, A_t, \theta_t) \right] \nabla \hat{q}(S_t, A_t, \theta_t) \end{aligned}$$

True value $q_\pi(S_t, A_t)$ generally unknown; use an unbiased estimate $U_t = R_t + \gamma \hat{q}(S_{t+1}, A_{t+1}, \theta)$ instead:

$$\theta_{t+1} = \theta_t + \alpha \left[U_t - \hat{q}(S_t, A_t, \theta_t) \right] \nabla \hat{q}(S_t, A_t, \theta_t)$$

Replace update to Q with update to θ

Example – Semi-gradient SARSA

Initialize $\theta \in \mathbb{R}^d$ arbitrarily

Loop for each episode:

  Initialize S_0

  Choose A as a function of $\hat{q}(S_0, ., \theta)$ e.g., ϵ-greedy

  Loop for each step of episode:

    Take action A, observe R, S'

    Choose A' as a function of $\hat{q}(S', ., \theta)$ e.g., ϵ-greedy

    $\theta \leftarrow \theta + \alpha[R + \gamma \hat{q}(S', A', \theta) - \hat{q}(S, A, \theta)]\nabla \hat{q}(S, A, \theta)$

    $S \leftarrow S'; A \leftarrow A'$

  until S is terminal

"The Deadly Triad"

Instability and **Divergence** arise when combining all three elements:

- **Function approximation**: Generalizing from a state space using linear functions or neural networks
- **Bootstrapping**: Targets include existing estimates (e.g. SARSA) rather than actual rewards only (e.g. MC methods)
- **Off-policy training**: Training on a distribution of transitions other than that produced by the target policy

DQN – Double Q Network

Experience Replay

- Store sequences S, A, R, S', A' in *replay buffer*
- FIFO queue of limited size
- Sample from replay buffer for each batch
- Removes/limits correlation of states within batches
- Smoothes data distribution changes

Target Network

- Maintain stable targets during updates
- Periodic update from ”main” network

DQN – Algorithm

Init replay buffer $D \leftarrow \emptyset$

Init main action-value function approximation $\hat{q}_M$ with random parameters θ_M

Init target action-value function approximation $\hat{q}_T$ with paramaters $\theta_T = \theta_M$

Loop for each episode:

  Initialize S

  For each step of the episode:

    Select action A using an ϵ-greedy policy based on $\hat{q}_M$

    Take action A and observe R, S_{t+1}

    Store transition (S_t, A_t, R_t, S_{t+1}) in D

    Sample minibatch (S_j, A_j, R_j, S_{j+1}) from D

    Target $y_j \leftarrow \begin{cases} r_j & \text{if } S_{j+1} \text{ is terminal} \\ r_j + \gamma \max_{A'} \hat{q}_T(S_{j+1}, A'; \theta^-) & \text{otherwise} \end{cases}$

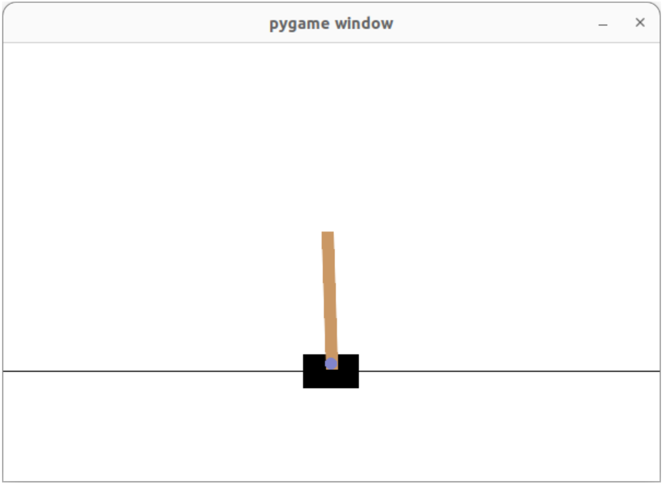
    $\theta \leftarrow \theta + \alpha[y_j - \hat{q}_M(S_j, A_j, \theta_M)]\nabla \hat{q}_M(S_j, A_j, \theta_M)$

  Every C steps, update $\hat{q}_T \leftarrow \hat{q}_M$ by setting $\theta_T \leftarrow \theta_M$

DQN – Algorithm

- In practice, S is a function $\phi(X)$ of inputs X through feature-extraction and pre-processing
- In practice, the update $y_j - Q(S_j, a_j, \theta)$ is clipped to $[-1, 1]$

DQN Example – Cart Pole



DQN Example – Cart Pole

Action Space:

0	Push cart to the left
1	Push cart to the right

State/feature space:

Num	*Observation*	*Min*	*Max*
0	Cart position	-4.8	4.8
1	Cart velocity	-Inf	Inf
2	Pole angle	-24 deg	24 deg
3	Pole angular velocity	-Inf	Inf

Rewards are +1 for every step taken

Termination occurs either:

- Pole angle is greater than ± 12 deg
- Cart position is greater than ± 2.4
- Episode length is grater than 200

DQN in Python

Use the ”CartPole” environment from `https://gymnasium.farama.org/`:

```python
import math
import random
import keras
from keras import layers
import gymnasium as gym
import tensorflow as tf
import numpy as np
import pygame

env = gym.make("CartPole-v1", render_mode="human")

Actions = range(0, env.action_space.n)
Ssize = env.observation_space.shape[0]
```

DQN in Python [cont'd]

Neural network and RL hyperparameters:

```
# Neural net parameters
batch_size = 8
dropout = 0.25
activation = 'relu'

epsilon = 0.8 # initial epsilon
gamma = 0.9 # discount factor
neps = 1000 # dicreasing epsilon factor
C = 50*batch_size # When to update weights

# Replay buffer D
D = collections.deque(maxlen=5000)
```

DQN in Python [cont'd]

Define the neural networks:

```
# Main network, used to select actions
Q = keras.Sequential([
    layers.InputLayer(input_shape=(Ssize+1),
                      batch_size=batch_size,
                      dtype=tf.float32),
    layers.Dense(Ssize*4, activation=activation),
    layers.Dropout(rate=dropout),
    layers.Dense(Ssize*2, activation=activation),
    layers.Dropout(rate=dropout),
    layers.Dense(1, activation='linear')
])
Q.compile(loss='huber', optimizer='adam')

# Target network, used to compute targets
Qhat = keras.models.clone_model(Q)
Qhat.compile(loss='huber', optimizer='adam')
Qhat.set_weights(Q.get_weights())
```

DQN in Python [cont'd]

Getting a $Q(s, a)$ value involves predicting from the neural net:

```
def getQ(Q, s, a):
    return Q.predict(np.expand_dims(np.array( \
        s.tolist()+[a]), axis=0), verbose=0)[0][0]
```

Max/Argmax operator for $Q(s, a)$ using prediction from main or target network:

```
def maxQ(Q, s, arg):
    maxq = -np.inf
    maxa = None
    for a in Actions:
        q = getQ(Q, s, a)
        if q > maxq:
            maxq = q
            maxa = a
    return maxa if arg else maxq
```

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DQN in Python [cont'd]

ϵ-greedy policy $pi_\epsilon(s)$

```
def pi(s, epsilon):
    if random.random() < epsilon:
        return random.choice(Actions)
    else:
        return maxQ(Q, s, True)
```

Update target for DQN:

```
def target_DQN(Q, Qhat, a, r, sprime):
    return r + gamma * maxQ(Qhat, sprime, False)
```

Update target for DDQN:

```
def target_DDQN(Q, Qhat, a, r, sprime):
    return r + gamma * getQ(Qhat, sprime, \
                            maxQ(Q, sprime, False))
```

Creating x and y data for training:

```
def training_xy(batch, ddqn=False):
    x = np.zeros((batch_size, Ssize+1))
    y = np.zeros(batch_size)
    for i, (s, a, r, t, sprime) in enumerate(batch):
        x[i] = list(s) + [a]
        if t == 1:
            y[i] = r
        else:
            if ddqn:
                y[i]=target_DDQN(Q, Qhat, a, r, sprime)
            else:
                y[i]=target_DQN(Q, Qhat, a, r, sprime)
    return x, y
```

DQN in Python [cont'd]

DQN/DDQN code:

```
for t in range(max_steps):
    s = env.reset()[0]
    a = pi(s, epsilon*math.exp(-t/neps))
    sprime, r, terminal, _, _ = env.step(a)
    G += r
    D.append((s, a, r, int(terminal), sprime))
    s = sprime
    if t >= batch_size:
        batch = random.sample(D, batch_size)
        x, y = training_xy(batch, ddqn=True)
        loss = Q.train_on_batch(x=x, y=y)

    if t % C == 0:
        Qhat.set_weights(Q.get_weights())
```

Complete code at
`https://evermann.ca/busi4720/DDQN_tuples.py`

DQN Extensions

Prioritized Experience Replay (PEX)

- Important actions are sampled with higher probability
- Use absolute TD error as priorities
- Faster learning

Double DQN

- Based on Double Q-Learning
- Uses target network $\hat{Q}$ as second Q function
- Removes upwards bias from using *max()* functions as estimator

Advantage Function

$$A(s, a) = Q(s, a) - V(s)$$

Advantage of action a in state s over the average action in state s

Rewrite as:

$$Q(s, a) = V(s) + A(s, a)$$

Dueling DQN

- Neural network from features x for value function $V(s)$ ("value stream")
- Neural network from features x for advantage function $A(s, a)$ ("advantage stream")
- Typically, value stream and advantage stream follow one or more common layers
- Aggregate to compute $Q(s, a)$

$$Q(s, a, \theta, \alpha, \beta) = V(s, \theta, \beta) + \left(A(s, a, \theta, \alpha) - \frac{a}{|\mathcal{A}|} A(s, a', \theta, \alpha) \right)$$

Where θ are shared neural-network parameters, β are parameters only for the "value-stream" neural network, and α are parameters only for the "advantage-stream" neural network

Policy Gradient Methods

Idea

Learn a parameterized policy :

$$\pi(s, a) = \pi(s, a, \theta) = \Pr(A_t = a | S_t = s, \theta_t = \theta)$$

Optimize:

$$J(\theta) = v_{\pi_\theta}(s_o)$$

Policy Gradient Methods

Advantages

- Simpler to approximate than action-value function
- Selection of actions with arbitrary probabilities
- Can better approach deterministic policy than ϵ-greedy action selection over action values
- Suitable for large and continuous action spaces

Policy Gradient Methods

REINFORCE update:

$$\theta_{t+1} = \theta_t + \alpha G_t \frac{\nabla \pi(A_t|S_t, \theta)}{\pi(A_t|S_t, \theta_t)}$$

- Update proportional to return G_t
- Update inversely proportional to action probability π

Policy Gradient Methods

REINFORCE: Monte-Carlo Control (episodic)

Input: A differentiable policy $\pi(a|s, \theta)$; step size $\alpha > 0$

Initialize policy parameters $\theta \in \mathbb{R}^d$ arbitrarily

Loop forever (for each episode):

Generate an episode $S_0, A_0, R_1, \ldots S_{T-1}, A_{T-1}, R_T,$

Loop for each step of the episode $t = 0, 1, \ldots, T-1$:

$$G \leftarrow \sum_{k=t+1}^{T} \gamma^{k-t-1} R_k$$

$$\theta \leftarrow \theta + \alpha G \nabla \ln \pi(A_t|S_t, \theta)$$

Policy Gradient Methods

REINFORCE with a baseline $b(S_t)$:

$$\theta_{t+1} = \theta_t + \alpha(G_t - b(S_t))\frac{\nabla\pi(A_t|S_t,\theta)}{\pi(A_t|S_t,\theta_t)}$$

Choose $b(S_t) = \hat{v}(S_t)$ the state-value function:

$$\theta_{t+1} = \theta_t + \alpha(G_t - \hat{v}(S_t))\frac{\nabla\pi(A_t|S_t,\theta)}{\pi(A_t|S_t,\theta_t)}$$

- Baseline leaves the expected value unchanged (unbiased)
- Can significantly reduce the variance
- Can improve speed of learning

REINFORCE with Baseline (episodic)

Input: A policy $\pi(a|s,\theta)$; step size $\alpha_\theta > 0$

Input: A state-value function $\hat{v}(s,w)$; step size $\alpha_w > 0$

Initialize parameters $\theta \in \mathbb{R}^d$, $w \in \mathbb{R}^d$ arbitrarily

Loop forever (for each episode):

Generate an episode $S_0, A_0, R_1, \ldots S_{T-1}, A_{T-1}, R_T$,

Loop for each step of the episode $t = 0, 1, \ldots, T-1$:

$$G \leftarrow \sum_{k=t+1}^{T} \gamma^{k-t-1} R_k$$

$$\delta \leftarrow G - \hat{v}(S_t, w)$$

$$w \leftarrow w + \alpha_w \delta \nabla \hat{v}(S_t, w)$$

$$\theta \leftarrow \theta + \alpha_\theta \gamma^t G \nabla \ln \pi(A_t|S_t,\theta)$$

One-Step Actor-Critic:

$$
\begin{aligned}
\theta_{t+1} &= \theta_t + \alpha(G_t - \hat{v}(S_t))\frac{\nabla\pi(A_t|S_t,\theta)}{\pi(A_t|S_t,\theta_t)} \\
&= \theta_t + \alpha(R_{t+1} + \gamma\hat{v}(S_{t+1},w) - \hat{v}(S_t,w))\frac{\nabla\pi(A_t|S_t,\theta)}{\pi(A_t|S_t,\theta_t)} \\
&= \theta_t + \alpha\delta_t\frac{\nabla\pi(A_t|S_t,\theta)}{\pi(A_t|S_t,\theta_t)}
\end{aligned}
$$

- Analogous to TD, SARSA and Q-Learning for tabular methods
- Improve on slow learning of MC methods
- Useful for non-episodic, continuous problems

Policy Gradient Methods

One-Step Actor-Critic

Input: A policy $\pi(a|s,\theta)$; step size $\alpha_\theta > 0$

Input: A state-value function $\hat{v}(s, w)$; step size $\alpha_w > 0$

Initialize parameters $\theta \in \mathbb{R}^d$, $w \in \mathbb{R}^d$ arbitrarily

Loop forever (for each episode):

  Initialize S (first state of episode); $I \leftarrow 1$

  Loop while S not terminal (for each time step):

    Sample A from $\pi(.|S,\theta)$

    Take action A, observe S', R

    $\delta \leftarrow R + \gamma\hat{v}(S', w) - \hat{v}(S, w)$

    $w \leftarrow w + \alpha_w \delta \nabla \hat{v}(S_t, w)$

    $\theta \leftarrow \theta + \alpha_\theta G \nabla \ln \pi(A_t|S_t,\theta)$

    $S \leftarrow S'$; $I \leftarrow \gamma I$

Reference Implementations

Stable Baselines

- Reference Python implementation of RL algorithms
- Pre-trained agents ("Baselines Zoo")
- Originally developed at OpenAI
- Pointers to additional learning materials

https://stable-baselines.readthedocs.io/en/master/

Gymnasium

- A standard programming interface (API) for RL environments
- Collection of reference environments
- Originally developed at OpenAI

https://gymnasium.farama.org/index.html

AlphaGo

https://www.alphagomovie.com

Google’s DeepMind division became famous in 2017 when it trained a computer to beat the human world champion at the game of Go. An award-winning full-length documentary has been made about this achievement.

https://www.alphagomovie.com
https://www.youtube.com/watch?v=WXuK6gekU1Y

The introductory paper by David Silver and others in Nature should be easy to understand: ”Mastering the game of Go without human knowledge”. Nature. 550 (7676): 354–359
https://www.nature.com/articles/nature24270

Additional Materials I

David Silver, UCL and Google DeepMind

Dr. Silver (`https://www.davidsilver.uk/`) of University College London has an excellent introductory course on reinforcement learning with class materials (from 2015) and lectures in a YouTube playlist. Updated courses (2018, 2021) are available on the DeepMind YouTube channel. The 2021 course include topics on deep reinforcement learning.

`https://www.davidsilver.uk/teaching/`

`https://www.youtube.com/playlist?list=PLqYmG7hTraZDM-OYHWgPebj2MfCFzFObQ`.

`https://www.youtube.com/@Google_DeepMind/playlists`.

Additional Materials II

UC Berkeley

UC Berkeley hosted a Deep RL Bootcamp in 2017 with slides and lecture videos available online. Additionally, UC Berkeley's course on Deep RL is available online, with lecture slides and videos of past years.

`https://sites.google.com/view/deep-rl-bootcamp/lectures`

`https://rail.eecs.berkeley.edu/deeprlcourse/`

Denny Britz

Formerly at the Google AI team, Denny Britz applied RL algorithms to financial markets and trading. He has a interesting blog, and a GitHub repository with resources and algorithm implementations of popular RL algorithms.

`https://dennybritz.com/`
`https://github.com/dennybritz/reinforcement-learning`

Additional Materials III

Massimiliano Patacchiola, Cambridge University

Dr. Patacchiola is a postdoc at Cambridge University. He has written a series of excellent blog posts on reinforcement based on the book "Artificial Intelligence – A Modern Approach" by Russell and Norvig. There are lots of illustrations and pointers to implementation and code in multiple languages.

`https://github.com/mpatacchiola/dissecting-reinforcement-learning`

Pascal Poupart, University of Waterloo

Dr. Poupart has made available videos and all course materials for all lectures for a course on reinforcement learning at UWaterloo.

`https://www.youtube.com/playlist?list=PLdAoL1zKcqTXFJniO3Tqqn6xMBBL07EDc`

`https://cs.uwaterloo.ca/~ppoupart/teaching/cs885-spring18/schedule.html`

Additional Materials IV

Andrew Ng, Stanford University

Dr. Ng (`https://www.andrewng.org/`) has taught an introductory class on reinforcement learning, as part of a broader course on machine learning.

`https://www.youtube.com/watch?v=RtxI449ZjSc`

`https://www.youtube.com/playlist?list=PLA89DCFA6ADACE599`

Andrei Karpathy, OpenAI, formerly Tesla

Andrei Karpathy (`https://karpathy.ai/` was a founding member of OpenAI (makers of ChatGPT and Dall-E) and later became the Tesla lead for their Autopilot program. An early blog post by Andrei Karpathy on RL is at the introductory level.

`https://karpathy.github.io/2016/05/31/rl/`

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Additional Materials V

Lilian Weng, OpenAI

Dr. Weng (`https://lilianweng.github.io/`) is a lead researchers at OpenAI (makers of ChatGPT and Dall-E). She has written an early blog post on RL and another one on policy gradient algorithms.

`https://lilianweng.github.io/posts/2018-02-19-rl-overview/`

`https://lilianweng.github.io/posts/2018-04-08-policy-gradient/`

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Additional Materials VI

OpenAI

OpenAI (https://openai.com; makers of ChatGPT and Dall-E) post regularly on their blog, on all things deep learning and also reinforcement learning. The blog posts are easy introduction to a variety of analytics topics.

https://openai.com/blog/openai-baselines-ppo/

https://openai.com/blog/evolved-policy-gradients/

https://openai.com/blog/evolution-strategies/

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